Math 10B with Professor Stankova
Worksheet, Discussion #3; Thursday, 1/31/2019
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Pigeonhole Principle

Concepts

1. Pigeonhole Principle gives us a guarantee on what can happen in the worst case scenario. The generalized principle says if N objects are placed into k boxes, then at least one box contains at least $\lceil N/k \rceil$ objects.

Examples

2. I have 7 pairs of socks in my drawer, one of each color of the rainbow. How many socks do I have to draw out in order to guarantee that I have grabbed at least one pair? What if there are likewise colored pairs of gloves in there and I cannot tell the difference between gloves and socks and I want a matching set?

3. Show that in a class of 30 students in 10B (consisting of freshmen, sophomores, juniors, and seniors), there exists at least 10 freshmen, 8 sophomores, 8 juniors, or 7 seniors.

Problems

4. True False The Pigeonhole Principle tells us that if we have $n+1$ pigeons and n holes, since $n+1 > n$, each box will have at least one pigeon.

5. True False The Pigeonhole Principle tells us that with n pigeons and k holes each hole can have at most $\lceil n/k \rceil$ pigeons.

6. Show that in a 8×8 grid, it is impossible to place 9 rooks so that they all don't threaten each other.

7. The population of the US is 300 million. Every person has written somewhere between 0 and 10 million lines of code. What's the maximum number of people that we can say must have written the same number of lines of code?

8. Three people are running for student government. There are 202 people who vote. What is the minimum number of votes needed for someone to win the election?

9. There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, what is the minimum number of different rooms that will be needed?

10. Assuming that everything in the US (300 million people) identifies with male or female and has less than 10 children, show that there exist at least 3 people that have the same gender, number of children, three letter initials, and birthday.

11. (Challenge) Show that in a group of 20 people and friendship is mutual, show that there exist two people who have the same number of friends?

Permutations and Combinations

Concepts

12. Permutations are when the order in which we choose matters (e.g. we line people up). The formula for choosing k things out of a total of n is $P(n,k) = n(n-1)(n-2)\cdots(n-k+1) = \dfrac{n!}{(n-k)!}$.

    Combinations are when the order does not matter (e.g. choose people for a team). The formula for choosing k things out of a total of n is $C(n,k) = \dfrac{n(n-1)(n-2)\cdots(n-k+1)}{k!} = \dfrac{n!}{(n-k)!k!}$.

Examples

13. How many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other?

14. How many anagrams of MISSISSIPPI exist?

15. How many anagrams of BEAD exist so that the vowels appear all next to each other?

Problems

16. True False $P(n,k) = C(n,k) \cdot k!$

17. True False $P(n,k) = P(n, n-k)$.

18. How many anagrams of ROYZHAO exist so that the consonants appear next to each other (Y is a vowel)?

19. How many ways are there to choose a delegation out of 10 males and 10 females if the delegation is made up of 2 males and 3 females?

20. At a consultant mixer with 42 people, everyone shakes everyone else's hand exactly once. How many handshakes occur?

21. 3 different friends are splitting 9 different donuts amongst themselves equally so each person gets 3. How many ways are there to do this?

22. How many four digit numbers exist such that their digits are in strictly increasing order?

23. How many rectangle sub-boards with at least two rows and columns exist on a 8×8 chessboard?

24. (Challenge) There are 9 points on a circle and lines connect all pairs of points. At how many places inside the circle do these lines intersect?